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Earle W. Owen

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SUMMARY

Expressions are obtained for the currents and voltages in a pair of inductively coupled superconducting coils under two conditions: formation of a normal zone and during a change in the level of the current in one coil. A dump resistor of low resistance and a detector bridge is connected across each coil. Calculated results are given for the MFTF coils. The circuit equations during formation of a normal zone are nonlinear and time-varying, consequently, only a series solution is possible. The conditions during a change in current are more easily found. After the transient has died away, the voltages in the coil associated with the changing source are all self-inductive, while the voltages in the other coil are all mutually inductive.

INTRODUCTION

The Mirror Fusion Test Facility (MFTF) magnet consists of a pair of superconducting coils with substantial inductive coupling (1). The construction of inductively coupled superconducting coils introduces a number of new technological problems. One of these is the detection of a normal zone. The purpose of this report is to provide information on the currents and voltages in the coils that will guide the designer of the control and protective equipment. Solutions are presented which can apply to any pair of coupled superconducting coils. Later in the report, calculated values for the MFTF coils are given.

Two conditions are of concern: the voltages and currents during the formation of a normal zone and the currents and voltages when the current in a coil is being changed from one level to another.

Normal zones are known to grow with time at a rate that depends on the current in the coil. The equations that describe the currents and voltages in the two coupled coils are a pair of nonlinear differential equations with a time varying coefficient. A closed form solution does not exist for these equations. However, a series solution gives adequate results for the short period during which a normal zone must be detected.

The time during which the current in a coil is being changed from one level to another is a critical period. During this time the mutual and self-induced voltages in the coils are large compared with the voltage across the normal zone. There are two methods of changing the current: by controlling the current of the source and by introducing a voltage. Both methods are discussed. However, emphasis is given to the method proposed for the MFTF magnet, the control of the source current.

It is assumed that a normal zone is detected by bridges formed by connecting resistors across the tapped coils. The detection circuits themselves are not discussed in detail. However, the operation of the bridges are included in the analysis.

THE EQUIVALENT CIRCUIT OF THE MAGNET

The superconducting magnet consists of two similar coils, disconnected electrically but coupled magnetically as shown in Figure 1. The resistors R_1 and R_2 are high power, low resistance resistors, permanently connected across the coils. They protect the magnet against an accidental open circuit in the power supply. They can also be used, as is the case for the MFTF magnets, to dissipate the magnet's energy internally in the event of a quench. The resistors are connected to ground near their midpoints in order to reduce the maximum voltage to ground during a quench.

The major portion of this report is concerned with similar coils, tapped at similar points. This means that the self-inductance and partial self-inductance of the two coils are the same.

There are several mutual inductances. The mutual inductance M_t is the mutual inductance from the top of one coil to the entire second coil. Because the two taps are at similar points on each coil and since the coils are identical to manufacturing tolerances, M_t is the same from either coil. Similarly, M_b is the mutual inductance from the bottom of either coil to the entire second coil.

Even when the coils are center tapped, the two partial mutual inductances M_t and M_b are not the same. This is because the two portions of a coil, the top half and the bottom half, have a different spacial orientation with respect to the other coil.

The Voltages in the Coils and Detection Circuits

The normal zone detector circuits consist of one or two bridges connected across each coil (2). Figure 2 shows the circuit with a single bridge connected across each coil. A number of voltages can be measured as inputs to the detector. They are v_1 and v_2 the coil voltages, v_L and v_R the bridge voltages, and v_T, v_C, and v_B, the voltages between the two coils.

The voltages in the circuit can be calculated from the current in the two coils. In this section the expressions are written down. These expressions are used for the specific cases discussed in the following section.

It is assumed that

$$L_1 = L_2 = L \text{ , } L_{t1} = L_{t2} = L_t \text{ , } L_{b1} = L_{b2} = L_b \text{ ,}$$

and $$R_1 = R_2 = R.$$

Compared with the current through the dump resistor R, the current through the bridges is negligible. For a self-inductance bridge the bridge resistors have the ratio

$$\frac{R_b}{R_B} = \frac{L_b}{L} \qquad \text{where } R_B = R_b + R_t.$$

For a mutual inductance bridge the ratio is

$$\frac{R_b}{R_B} = \frac{M_b}{M}.$$

The voltages in the left coil are

$$v_{ac} = L_t \frac{di_1}{dt} + M_t \frac{di_2}{dt} ,$$

$$v_{cd} = L_b \frac{di_1}{dt} + M_b \frac{di_2}{dt} ,$$

$$v_{de} = i_1 r \ (t) .$$

The total voltage is

$$v_1 = v_{ac} + v_{cd} + v_{de} ,$$

or

$$v_1 = (I_1 - i_1) \ R_1 .$$

The voltages across the bridge resistors are

$$v_{ab} = \frac{R_{t1}}{R_{B1}} v_1 ,$$

$$v_{be} = \frac{R_{b1}}{R_{B1}} v_1 .$$

The output of the bridge is

$$v_L = v_{ce} - v_{be} .$$

The voltages in the right coil are

$$v_{fg} = L_t \frac{di_2}{dt} + M_t \frac{di_1}{dt} ,$$

$$v_{gj} = L_b \frac{di_2}{dt} + M_b \frac{di_1}{dt} ,$$

and

$$v_2 = v_{fg} + v_{gj} .$$

The voltages across the bridge resistors are

$$v_{fh} = \frac{R_{t2}}{R_{B2}} \ v_2 ,$$

$$v_{hj} = \frac{R_{b2}}{R_{B2}} \ v_2 ,$$

and the output of the bridge is

$$v_R = v_{gj} + v_{jh} .$$

One input to the detector is the difference between the two bridge voltage, $v_R - v_L$ another is $v_C - v_B$, or $v_C - v_T$. The voltage $v_C - v_B$ is, $v_C - v_B = v_{cd} + v_{de} - v_{jg}$.

CONDITIONS DURING THE FORMATION OF A NORMAL ZONE

The Growth of a Normal Zone

If a small normal zone forms in a superconducting coil, it will either grow, if the heat generated is sufficient, or shrink, restoring the coil to its superconducting state(3). The growth of the zone has been investigated both theoretically and experimentally. The description of normal zone growth used in this report is based on the work of Smith (4). According to this development,

$$r(t) = \frac{mL}{I}\, i(t)\, t$$

$r(t)$	=	the resistance of the normal zone	(Ω)
t	=	time, when growth begins at $t = 0$	(s)
I	=	the initial current, at $t = 0$	(A)
$i(t)$	=	the instantaneous current	(A)
L	=	the self-inductance of the coil	(H)
m	=	the rate of growth factor	(s^{-2})

The validity of this theoretical model has been verified experimentally. In particular, tests on a large experimental coil at the Lawrence Livermore Laboratory have demonstrated a similar growth from an artifically induced normal zone (5). It has also been shown that the normal zone propagates from coil to coil as well as along the coils, a fact not accounted for by the theoretical model. However, during the initial stages of growth, when detection is vital, the simple model is adequate.

Currents During Formation of a Normal Zone

An equivalent circuit, suitable for the analysis of the growth of a normal zone is shown in Figure 3. The current to the bridge resistors is small and can be neglected. The normal zone is arbitrarily placed in the bottom of the left coil.

The circuit equations are

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + i_1\, r(t) + (i_1 - I_1)\, R = 0 \ ,$$

$$L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} + (i_2 - I_2)\, R = 0 \ .$$

The solution is simplified without handicapping its utility if the currents in the two magnets are equal

$$I_1 = I_2 = I \ .$$

In addition, the circuit has been at rest for a long time so that,

$$i_1\,(0) = i_2\,(0) = I \ .$$

The normal zone grows in the manner described in the previous section,

$$r(t) = \frac{mL}{I}\, t\, i_1 \,.$$

Therefore, the equations are

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + \frac{mL}{I}\, t\, i_1^2 + R_1\, i_1 = IR_1 \,,$$

$$M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \qquad + R_2 i_2 = IR_2 \,.$$

Because of the term $\frac{mL}{I}\, t\, i_1^2$ the set of equations is nonlinear, and in addition, has a time-varying coefficient. There is no closed form solution of this equation, but a series solution can be obtained. This solution adequately describes the conditions during the initial growth of the normal zone, the period of time of particular interest.

Let the series solution be

$$i_1 = I\,(1 + a_1 t + a_2 t^2 + a_3 t^3 + \ldots) \,,$$

$$i_2 = I\,(1 + b_1 t + b_2 t^2 + b_3 t^3 + \ldots) \,.$$

Substituting these solutions into the differential equations results in the two algebraic equations

$$L_1 I\,(a_1 + 2a_2 t + 3a_3 t^2 + \ldots) + MI\,(b_1 + 2b_2 t + 3b_3 t^2 + \ldots)$$

$$+ mLtI\left[1 + 2a_1 t + (2a_2 + a_1^2)\, t^2 + \ldots\right] + R_1 I\,(1 + a_1 t + a_2 t^2 + \ldots) = IR_1 \,,$$

and

$$L_2 I\,(b_1 + 2b_2 t + 3b_3 t^2 + \ldots) + MI\,(a_1 + 2a_2 t + 3a_3 t^2 + \ldots)$$

$$+ R_2 I\,(1 + b_1 t + b_2 t^2 + \ldots) = IR_2 \,.$$

The coefficients of the series are evaluated by equating the terms containing similar powers of t. The two equations yield a pair of equations that can be solved simultaneously for the a's and b's. Equating the coefficients of t^0 gives the two equations,

$$L_1 I a_1 + MI\, b_1 = R_1\,(I - I),$$

$$MI a_1 + LI\, b_1 + R_2\,(I - I).$$

Therefore, $\quad a_1 = b_1 = 0 \,.$

Equating the coefficients of t^1 gives,

$$2L_1 a_2 + 2Mb_2 = -mL_1 \,,$$

$$2\,Ma_2 + 2L_2 b_2 = 0 \,.$$

Solving this set of equations yields

$$a_2 = -\frac{m}{2(1-K^2)},$$

$$b_2 = \frac{mM}{2L_2\,(1-K^2)}.$$

Where K is the coupling coefficient of the coils, $K = \frac{M}{\sqrt{L_1 L_2}}$.

Following a similar procedure,

$$a_3 = \frac{m}{6(1-K^2)^2} \quad \frac{1}{\tau_1} + \frac{K^2}{\tau_2},$$

$$b_3 = \frac{-mM}{6L_2\,(1-K^2)^2} \quad \frac{1}{\tau_1} + \frac{1}{\tau_2}.$$

where $\tau = \frac{L}{R}$.

Voltages During Formation of a Normal Zone

The voltages in the circuit during formation of a normal zone can now be written. Only the terms of the infinite series that are significant during the brief period during which the normal zone must be detected are included.

For the left coil

$$v_{ac} = I\,[2\,(L_t a_2 + M_t b_2)\,t + 3\,(L_t a_3 + M_t b_3)\,t^2],$$

$$v_{cd} = I\,[2\,(L_b a_2 + M_b b_2)\,t + 3\,(L_b a_3 + M_b b_3)\,t^2],$$

$$v_{de} = mLI\,(t + 2a_2 t^2),$$

$$v_1 = -RI\,(a_2 t^2 + a_3 t^3).$$

For the right coil

$$v_{fg} = I\,[2\,(M_t\,a_2 + L_t\,b_2)\,t + 3\,(M_t\,a_3 + L_t\,b_3)\,t^2],$$

$$v_{gj} = I\,[2\,(M_b\,a_2 + L_b\,b_2)\,t + 3\,(M_b\,a_3 + L_b\,b_3)\,t^2],$$

$$v_2 = I\,[2\,(Ma_2 + Lb_2)\,t + 3\,(Ma_3 + Lb_3)\,t^2],$$

but since $La_2 + Mb_2 = 0$

$$v_2 = 3I\,(La_3 + mb_3)\,t^2.$$

CONDITIONS DURING A CHANGE IN CURRENT

A critical period for the detection of a normal zone is during a change in the current from one level to another. During this period, the mutual and self-induced voltages are large compared with the voltages caused by a normal zone.

In the MFTF magnet the change in current is brought about by slowly changing the controlled current of the power supply in a ramp-like fashion. During this period, the power supply can be modelled by a time variable current source. An alternative method of changing the current is to introduce a voltage into the circuit. Analysis of this method is confined to Appendix B.

When a change in current is initiated, a transient is produced. However, since changing the current in a large magnet requires a long time, a steady state is soon reached. The steady state period is of special significance because, during this period, the normal zone detector can be adjusted and balanced.

The Steady State Current Caused by a Current Ramp

The circuit is the same as Figure 3 except that, for the moment, the normal zone does not exist

$$r(t) = 0 .$$

In addition, one of the current sources is increasing in a ramplike fashion

$$I_1 = I \quad 1 + ft \quad ,$$

$$I_2 = I .$$

Before the beginning of the ramp, it is assumed that the circuit has been quiescent for a long time.

Therefore,

$$i_1 (0) = i_2 (0) = I.$$

The circuit equations are

$$L \frac{di_1}{dt} + M \frac{di_2}{dt} + i_1 R = IR (1 + ft),$$

$$M \frac{di_1}{dt} + L \frac{di_2}{dt} + i_2 R = IR.$$

The equations is solved by a straightforward application of the Laplace transform in Appendix A. It is found that

$$\frac{i_1}{I} = ft + (1 - \frac{L}{R}) f + a e^{-t/\tau_a} + b e^{-t/\tau_b}$$

$$\frac{i_2}{I} = 1 - \frac{M}{R} f + d e^{-t/\tau_a} + g e^{-t/\tau_b}$$

The time constants τ_a and τ_b are both positive and real. After the transient dies away the steady state current is

$$i_{1s} = I\,[ft + (1 - \frac{L}{R})\,f]\,,$$

$$i_{2s} = I\,(1 - \frac{MI}{R}\,f).$$

The Steady State Voltages Caused by a Current Ramp

The voltages in the coils and detector circuits can be calculated using the expressions derived in a previous section. Figure 4 shows the results.

When steady state is reached all of the ramp current, Ift, goes through the left coil. The change in current generates a self-induced voltage, LfI, across the coil, and a corresponding current through the left dump resistor. The current through the bridge is relatively small and can be neglected.

The change in current causes a mutually induced voltage, Mft, to be generated in the right coil. The presence of this voltage causes a readjustment of the currents in the right circuit. Some of the fixed current from the source flows through the dump resistor. The current in the right coil is constant after the transient has died away. Therefore, there are no mutually induced voltages reflected back to the left coil.

An important observation to make is that, during steady state, all of the voltages in the side associated with the changing source are self-inductive, while those on the other side are mutually inductive. It is this property that makes the balancing of the bridges possible.

The Transient During a Current Ramp

The total solution of the circuit during a ramp change in the current source is given in Appendix A. Equations A1 and A2 show the total current, which is a sum of the steady state and transient currents. The transient portion is the part associated with the decaying exponentials.

The detection of a normal zone is not as onerous during the transient period as it is during the steady state. This conclusion can be justified by examining the rate of change of current in the left coil, on which both the mutual and self-induced voltages depend:

$$\frac{di_1}{dt} = fI\left[1 - \frac{1}{2}\;(e^{-t/\tau_a} + e^{-t/\tau_a})\right]\;.$$

The rate of change of current is zero at zero time and increases to the steady state value of fI as time increases. Therefore, the self-induced voltage in the left coil and the mutually induced voltages in the right coil are at a maximum during steady state. A more exact analysis would take into account the rate of change of the current in the left coil, $\frac{di_2}{dt}$. This derivative has a maximum during the transient period. However, the effect of this changing current is relatively small.

CALCULATED RESULTS FOR THE MFTF MAGNET

The Circuit Inductances and Resistances

In this section, calculated results are presented for the center tapped coils. The mutual and self-inductances are taken from a memo by D. Shimer, May 4, 1979.

The value of dump resistor is a proposed design value based on a maximum voltage of approximately 500 V during a quench.

$$L_b = 5.32 \text{ H}, \; L_t = 5.79 \text{ H}, \; L = L_b + L_t = 11.11 \text{ H},$$

$$M_b = .617 \text{ H}, \; M_t = .532 \text{ H}, \; M = M_b + M_t = 1.149 \text{ H},$$

$$R_1 = R_2 = R = .17 \quad .$$

Calculated Values During Normal Zone Formation

Calculations have been made for the core when the two dump resistors are identical. Therefore, $\tau_1 = \tau_2 = \frac{L}{R} = \frac{11.11}{.17} = 65.37$ s. The coupling coefficient, $K = \frac{M}{L}$, is .1034.

The coefficients of the series expressions for the currents are

$$a_2 = -\,.5054 \text{ m} \quad , \; a_3 = 2.630 \times 10^{-3} \text{ m} \; ,$$

$$b_2 = .05225 \text{ m} \; , \; b_3 = -5.420 \times 10^{-4} \text{ m}.$$

where m is a measure of the rate of growth of the normal zone, as described in an earlier section.

Several factors occur frequently in the calculation of the normal zone voltages. They are listed below

	$2(La_2 + Mb_2)$	$3(La_3 + Mb_3)$	$3(Ma_3 + Lb_3)$
Upper, $M \leftarrow M_t$ $L \leftarrow L_t$	-5.800 m	4.484×10^{-2} m	-5.222×10^{-3} m
Lower, $M \leftarrow M_b$ $L \leftarrow L_b$	-5.313 m	4.097×10^{-2} m	-3.782×10^{-3} m
Total, $M \leftarrow M$ $M \leftarrow L$	-11.113 m	8.581×10^{-2} m	-9.004×10^{-3} m

Calculations have been made for a growth of the normal zone equal to one meter per second. The resistance of one meter of normal conductor is taken to be 4.6×10^{-6} ohms. Therefore,

$$r(t) = 4.60 \times 10^{-6}\ t.$$

From the section on the growth of the normal zone,

$$r(t) = \frac{mL}{I}\ i(t)\ t\ .$$

At t=0, i(t) = I. Therefore

$$m = \frac{4.60 \times 10^{-6}}{L}\ I = 4.140 \times 10^{-7}\ I\ s^{-2}$$

The significant calculated results, for an initial current of 6000 A, are presented in Figure 5, 6, and 7. Figure 5 shows the voltages in the coil containing the normal zone. The growth of the normal zone causes a decrease in the current. The resultant self-induced voltage tends to maintain the current. Since the self-inductance is relatively large compared with the resistance of the dump resistor the self-induced voltage is large, almost as large as the voltage across the normal zone but in the opposite direction. Therefore, the voltage that appears across the magnet, v_1, is relatively small.

Figure 6 shows the current and the rate of change of current in the left circuit. The change is very small, only about 24 mA in 6000 A after 20 seconds of normal zone growth. Since the source supplies a constant current, the small decrease in coil current flows through the dump resistor.

Finally, Figure 7 shows the calculated voltages for the two coils and bridges. A time of 10 s, when the normal zone is 10 meters long, has been used in his presentation.

Calculated Values During a Steady State Increase in Current

Figure 4 shows the expression for the voltages throughout the circuit for steady state conditions when the coil current is being changed by a ramp change in the current source. A numerical evaluation of these expressions, for an increase in current of 1 As^{-1} from 6000 is shown in Figure 8.

Calculated Values for a Transient During a Change in Current

When the change in current is caused by a ramp change in the source, the two coils act as a coupled system. The two time constants are

$$\tau_a = \frac{L\ (1 + K)}{R} = \frac{11.11}{.17}\ (1 + .1034) = 72.11\ s$$

$$\tau_b = \frac{L\ (1-K)}{R} = 58.60\ s$$

The voltage across the coil associated with the change in current is

$$v_1 = LIf \left[1 - \frac{1}{2}\ (e^{-t/\tau_a} + e^{-t/\tau_b}) \right].$$

This voltage is plotted in Figure 9. The voltage begins at zero and exponentially approaches the steady state value of LIf.

CONCLUSIONS

A number of conclusions of general interest to the designer of inductively coupled superconducting coils can be made.

The formation of a normal zone causes very small changes in the currents of a pair of large superconducting coils. When the inductive coupling is small, the major effect is confined to the coil in which the normal zone occurs. Calculations of the voltages and currents during the first few seconds of formation of a normal zone show little difference between a coupled MFTF coil and one assumed to be uncoupled. (The coupling coefficient is about .1.)

During a change in current from one level to another, the mutually induced voltages are relatively large. Both the self-induced and mutually induced voltages are a maximum after the transient has died away and the increase in coil current is constant. Therefore, detection of the normal zone is more difficult during the steady state than it is during the transient.

During a steady state change in the current, the voltages in the coil associated with the changing supply are all self-inductive and the voltages in the other coil are all mutually inductive. This is true for the two ways of changing the coil current - by a current ramp or by a voltage source.

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APPENDIX A

The Transient and Steady State Currents Caused by a Ramp Current Source

In this section the currents in the two coils are determined during a ramp change in the current of one source. Both the transient and steady state conditions are found. The circuit equations are solved by conventional LaPlace transform methods.

Figure 3 shows the circuit. The left current source increases linearly beginning at t = 0 while the right current source is constant. Initially, both coils have the same current.

$I_1(t) = I\,(1 + ft)$, where f is the rate factor,

$I_2(t) = I.$

And,

$i_1\,(0) = i_2\,(0) = I.$

The circuit equations are:

$$L\,\frac{di_1}{dt} + M\,\frac{di_2}{dt} + i_2 R = I\,(1 + ft)\,R\ ,$$

$$M\,\frac{di_1}{dt} + L\,\frac{di_2}{dt} + i_2 R = IR\ .$$

Transforming, and writting in matrix form yield

$$\begin{bmatrix} Ls + R & sM \\ sM & Ls + R \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} I\quad L + M + R\,\left(\frac{1}{s} + \frac{f}{s^2}\right) \\ I\quad \left(L + M + \frac{R}{s}\right) \end{bmatrix}$$

The characteristic equation is

$$s^2 + \frac{2LR}{L^2 - M^2}\,s + \frac{R^2}{L^2 - M^2} = 0\ ,$$

which can be written as

$$\left[s + \frac{R}{L\,(1 + K)}\right]\left[s + \frac{R}{L\,(1 - K)}\right] = 0\ .$$

A pair of similar, inductively coupled coils is always overdamped with time constants,

$$\tau_a = \frac{L\,(1 + K)}{R}\ ,\ \text{and}\quad \tau_b = \frac{L\,(1 - K)}{R}.$$

K is the coupling coefficient,

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L}, \quad K < 1.$$

The current I_1 is equal to the ratio of the determinants:

$$\frac{I_1}{I}(s) = \frac{\begin{vmatrix} L + M + R\left(\frac{1}{s} + \frac{f}{s^2}\right) & sM \\ L + M + \frac{R}{s} & Ls + R \end{vmatrix}}{\begin{vmatrix} Ls + R & sM \\ sM & Ls + R \end{vmatrix}}$$

Evaluating and simplifying,

$$\frac{I_1}{I}(s) = \frac{1}{s} + \frac{Rf\,(s + R/L)}{L\,(1-k^2)\,s^2\,(s + \frac{1}{\tau_a})\,(s + \frac{1}{\tau_b})}\,.$$

The inverse transform is found by a partial fraction expansion,

$$\frac{I_1}{I}(s) = \frac{1}{s} + \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s + \frac{1}{\tau_a}} + \frac{D}{s + \frac{1}{\tau_b}}.$$

Evaluating the partial fraction coefficients and transforming gives

$$\frac{i_1}{I}(t) = ft + (1 - \frac{L}{R} f) + \frac{fL}{2R}\left[(1 + K)\,e^{-t/\tau_a} + (1 - K)\,e^{-t/\tau_b}\right]. \qquad \text{(A1)}$$

A similar procedure is used to solve for $i_2(t)$.

$$\frac{i_2}{I}(t) = 1 - \frac{Mf}{R} + \frac{Mf}{2RK}\left[(1 + K)\,e^{-t/\tau_a} - (1 - K)\,e^{-t/\tau_b}\right]. \qquad \text{(A2)}$$

The rates of change of currents are also of interest

$$\frac{di_1}{dt} = If\left[1 - \frac{1}{2}\left(e^{-t/\tau_a} + e^{-t/\tau_b}\right)\right].$$

$$\frac{di_2}{dt} = \frac{fM}{2KL} I \left(e^{-t/\tau_a} - e^{-t/\tau_b}\right).$$

APPENDIX B

The Transient and Steady State Currents Caused by a Voltage Source

The current in the coil can be increased by introducing a voltage into the circuit. In the MFTF coils this method is not used, the current is changed by a change in the controlled current of the supply. However, conditions that exist when a voltage is present are of sufficient interest to justify this brief Appendix.

The circuit model for a change in coil voltage is shown in Figure A1. A voltage source in parallel with the current source causes a change in the coil current. (A voltage source in series with it has no effect.)

When the switch in Figure A1 is closed, a voltage is imposed across the left coil causing a constant rate of change of current,

$$v_1 = V = L\frac{di_1}{dt} .$$

The two current sources are assumed to be equal, $I_1 = I_2 = I$.

Since the voltage across the left coil is fixed, the right coil has no effect on it. However, the changing current in the left coil induces a voltage in the right coil. A first order differential equation describes the current in the right coil,

$$M\frac{di_1}{dt} + L\frac{di_2}{dt} + i_2R = IR.$$

But

$$\frac{di_1}{dt} = \frac{V}{L} .$$

Therefore,

$$\frac{di_2}{dt} + \frac{R}{L}\, i_2 = \frac{1}{L}\,(IR - KV),$$

where

$$K = \frac{M}{L} .$$

The solution of this equation is

$$i_2(t) = I - \frac{KV}{R}\left(1 - e^{\frac{-Rt}{L}}\right) ,$$

when,

$$i_2(0) = I.$$

The current in the left coil is

$$i_1 = I + \frac{V}{L}\, t.$$

After the transient has died away the steady state currents, i_{1s} and i_{2s}, are

$$i_{1s} = I + \frac{V}{L} t$$

$$i_{2s} = I - \frac{KV}{R}$$

The steady state voltages across the coils and bridges are shown in Figure A1.

The steady state voltages caused by a voltage source can be compared with the steady state voltages caused by a ramp change in current which are shown in Figure 4. In both cases there is a steadily changing current in the coil associated with the source causing the change and a constant current in the other coil. As a result, all of the voltages in the coil associated with the changing source are self-induced voltages, and all the voltages in the other coil are mutually induced voltages.

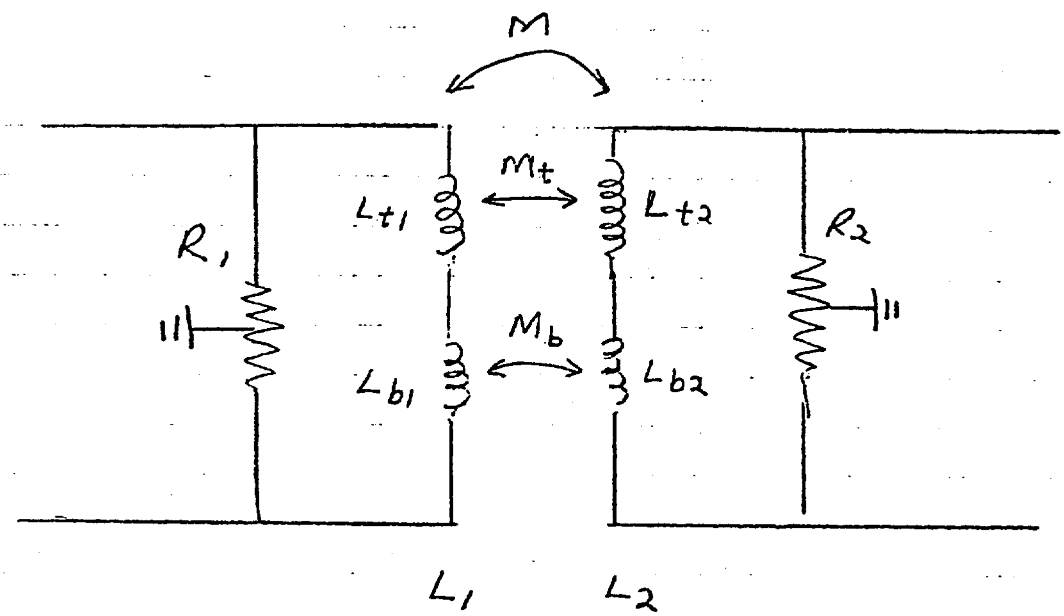

FIGURE 1

A pair of mutually coupled superconducting coils with permanently connected dump resistors

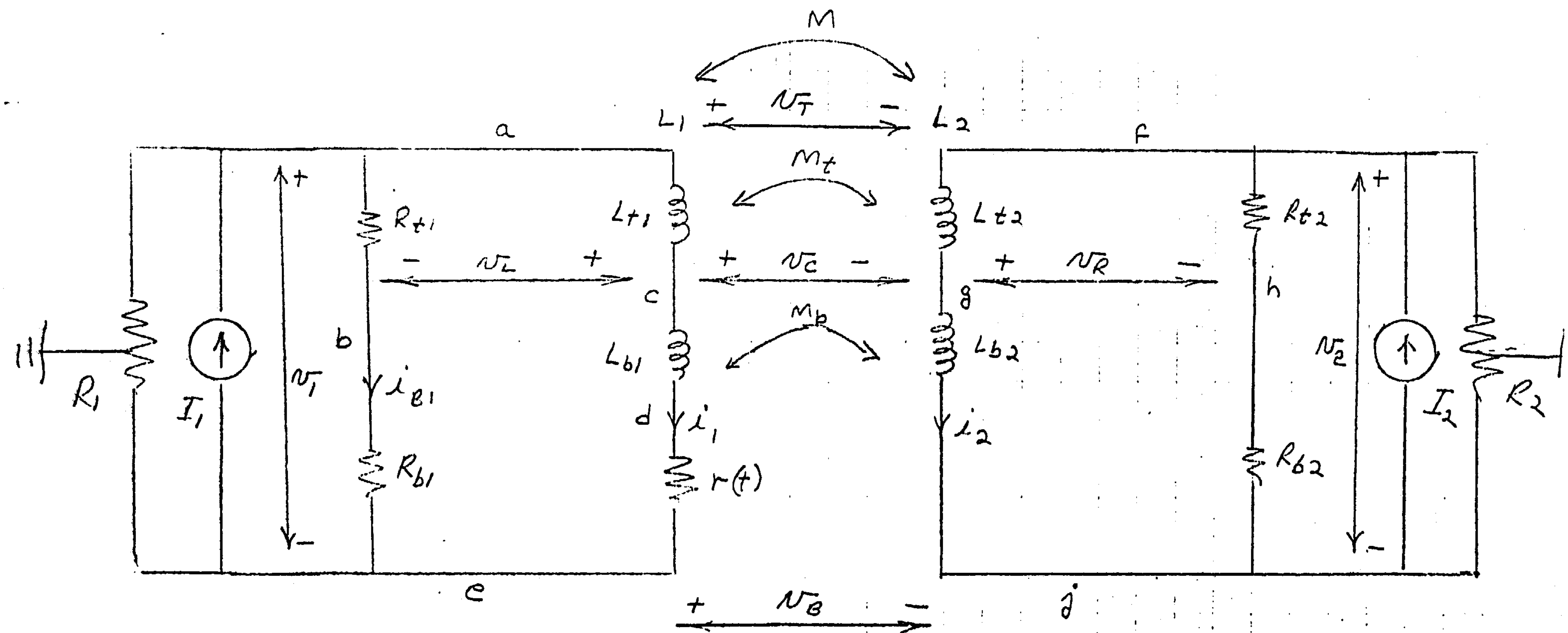

FIGURE 2

The superconducting coils with resistive bridges R_{b1}, R_{t1} and R_{b2}, R_{t2} connected across them. The normal zone resistance is $r(t)$.

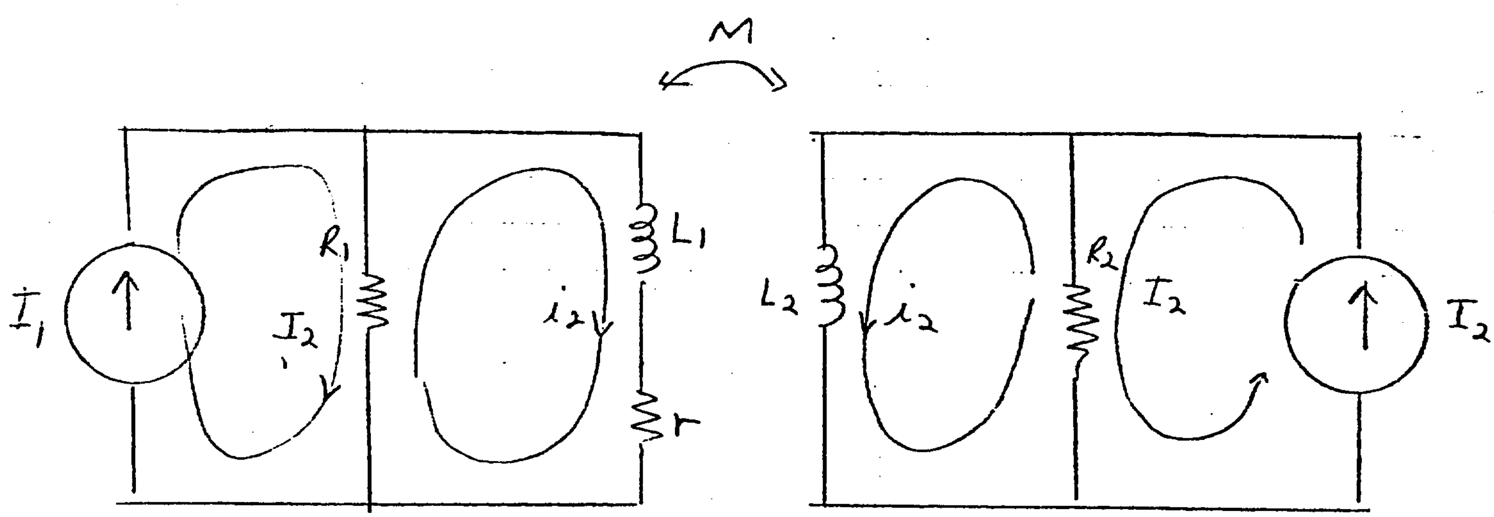

FIGURE 3

Loop currents in the coils driven by current sources

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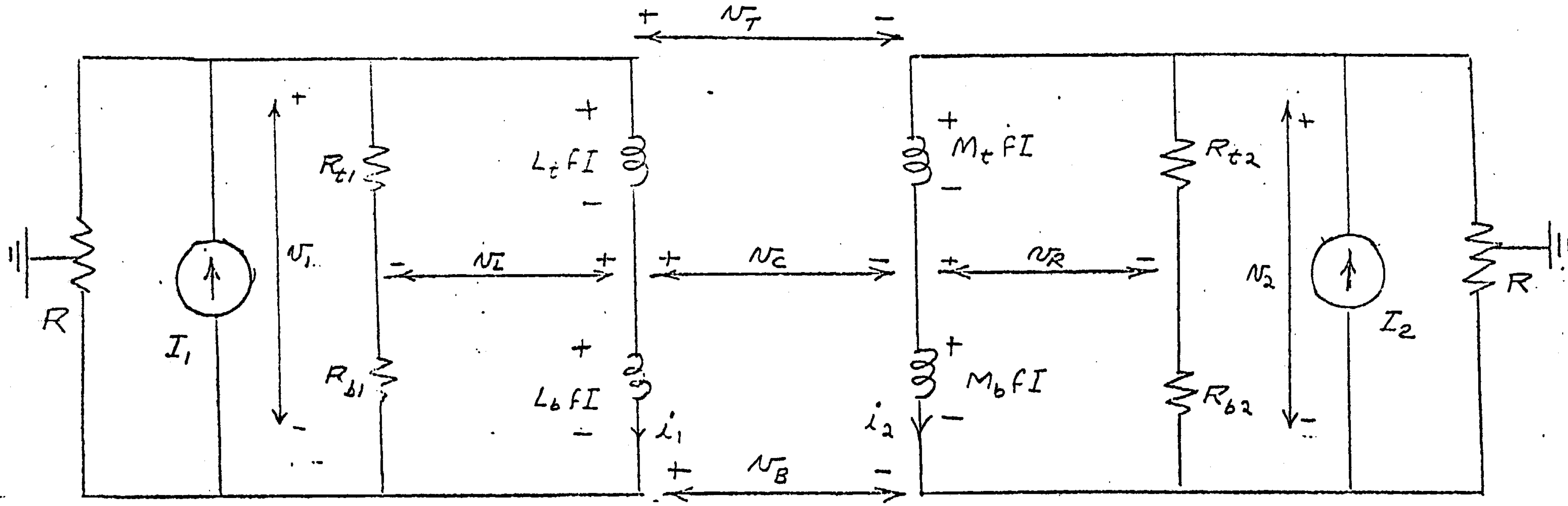


$I_1 = I(1+ft)$ $\quad$ $i_1 = I\left[(1-\frac{L}{R}f)+ft\right]$ $\quad$ $i_2 = I(1-\frac{M}{R}f)$ $\quad$ $I_2 = I$

$v_1 = +LfI$ $\quad$ $v_L = fIL\left(\frac{L_b}{L} - \frac{R_{b1}}{R_{B1}}\right)$ $\quad$ $v_C - v_B = fI(L_b - M_b)$

$v_2 = +MfI$ $\quad$ $v_R = fIM\left(\frac{M_b}{M} - \frac{R_{b2}}{R_{B2}}\right)$ $\quad$ $v_C - v_T = fI(M_t - L_t)$

FIGURE 4

Voltages and currents during a steady state increase of I_1

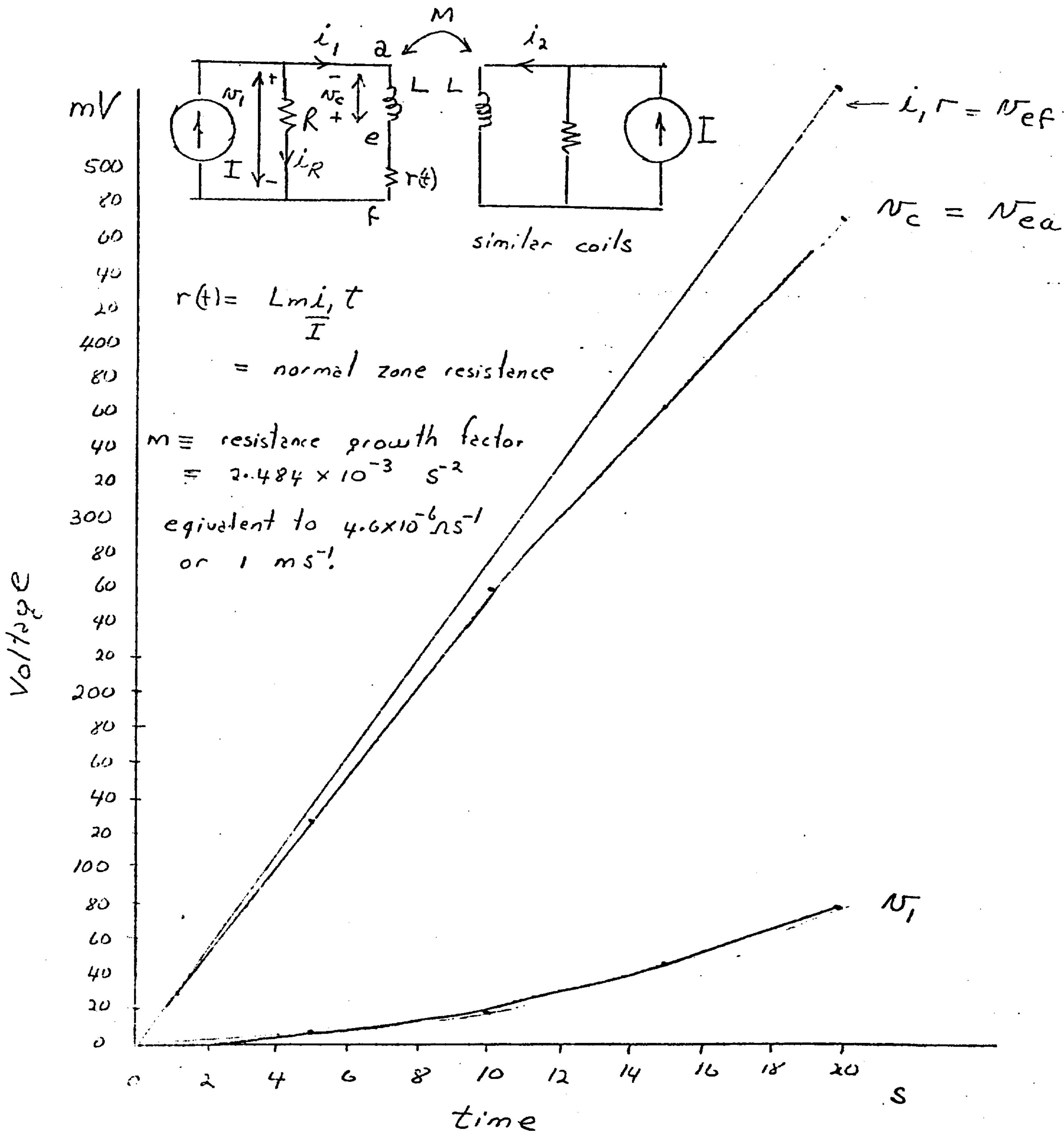


FIGURE 5

Voltages in the left MFTF coil during the formation of a normal zone. The normal zone is initially at 1 ms^{-1} or 4.6 μΩs^{-1}. This leads to a value for m of 4.4?? × 10^{-7} T s^2 I=???? A L=11.113 H M=1.1??

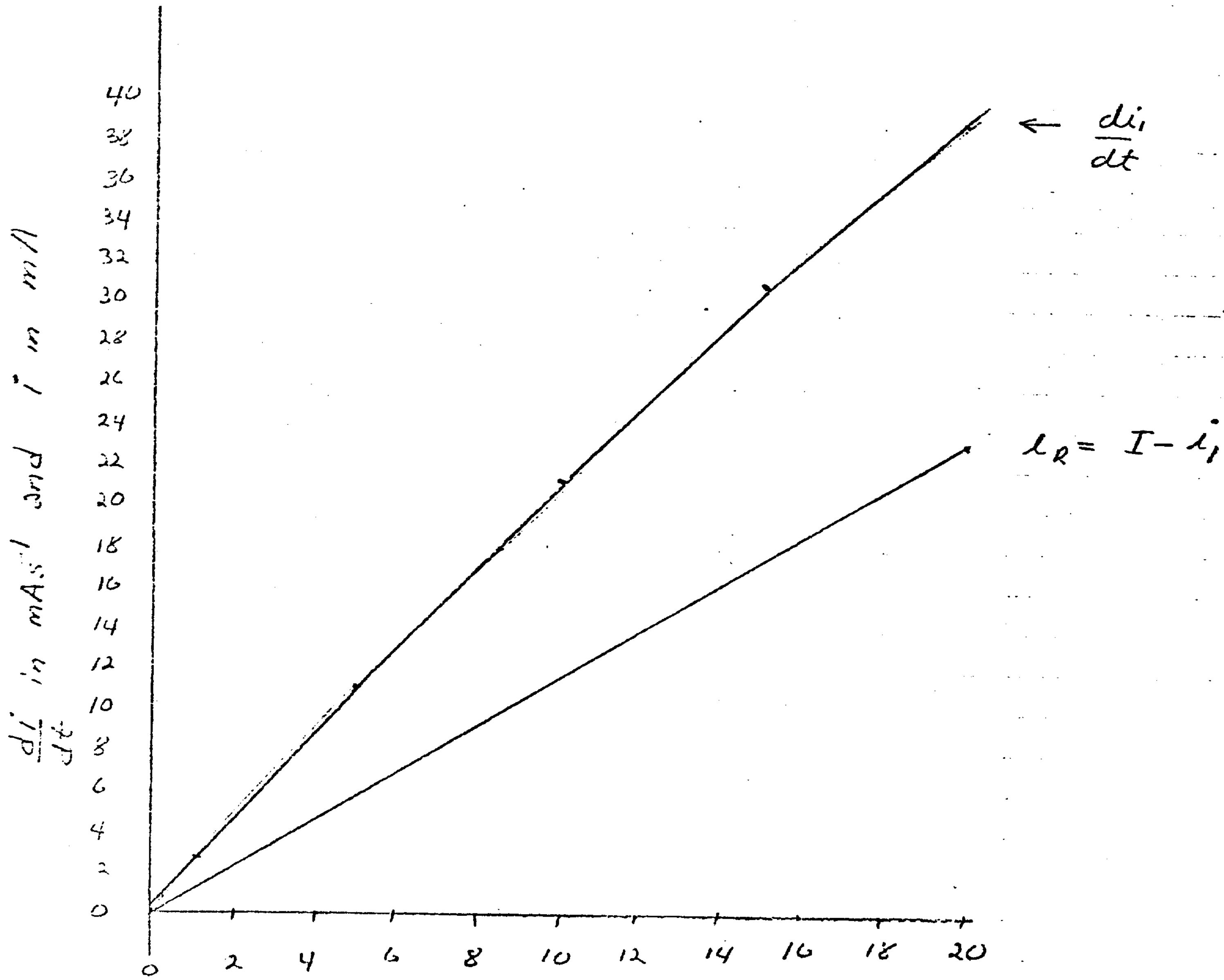
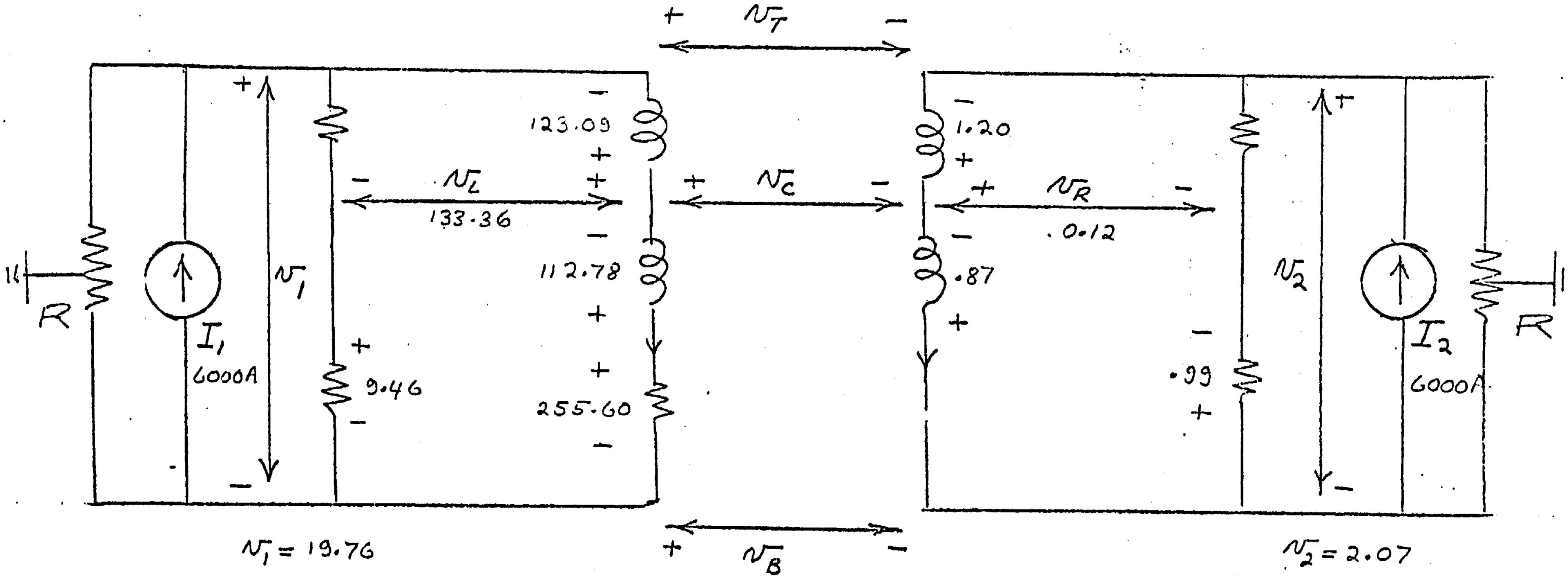


FIGURE 6

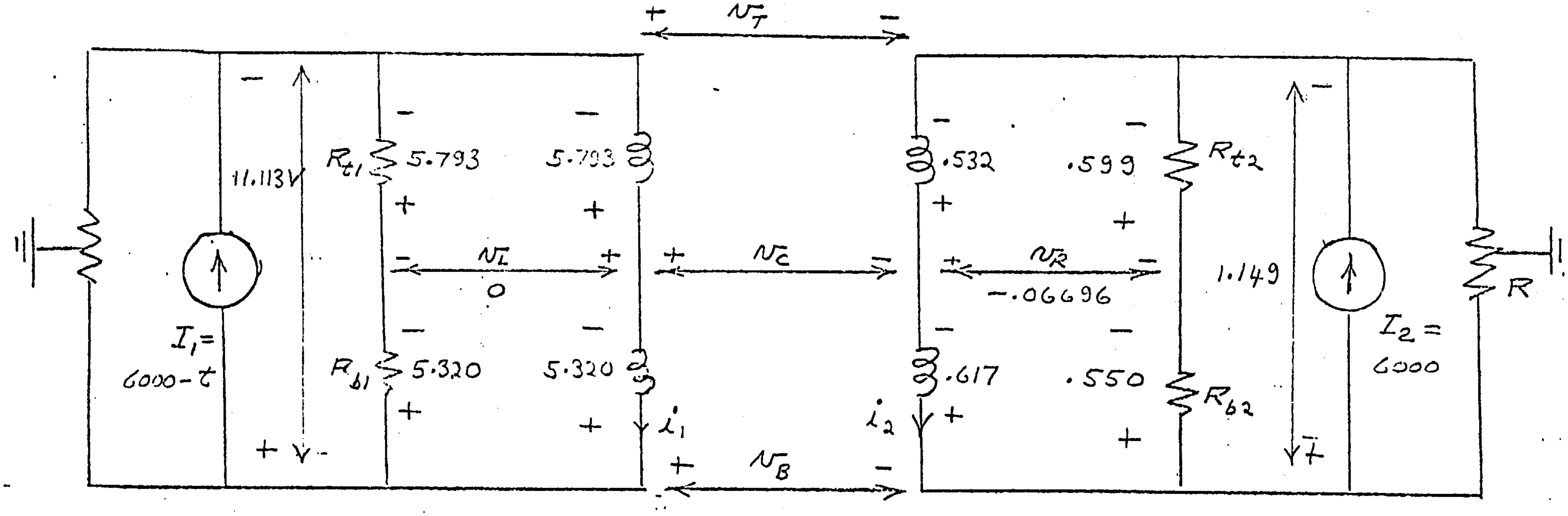
The current and rate of change of current in the left coil and circuit. The conditions are the same as in Figure 5.



$v_C - v_B = 143.69$

FIGURE 7

Voltages during growth of the normal zone, at $t = 10$ s. The conditions are the same as in Figure . The voltages are in millivolts. The bridges are balanced with the self-inductances, $\frac{R_b}{R_B} = \frac{L_b}{L}$.



$$v_c - v_B = -5.320 + .617 = -4.703$$

FIGURE 8

Voltages during a steady state decrease of I_1 of $1\,As^{-1}$ from 6000 A. The voltages are in volts. The two bridges have been balanced with the self-inductances, $\frac{R_b}{R_B} = \frac{L_b}{L}$. The coils are center-tapped and similar.

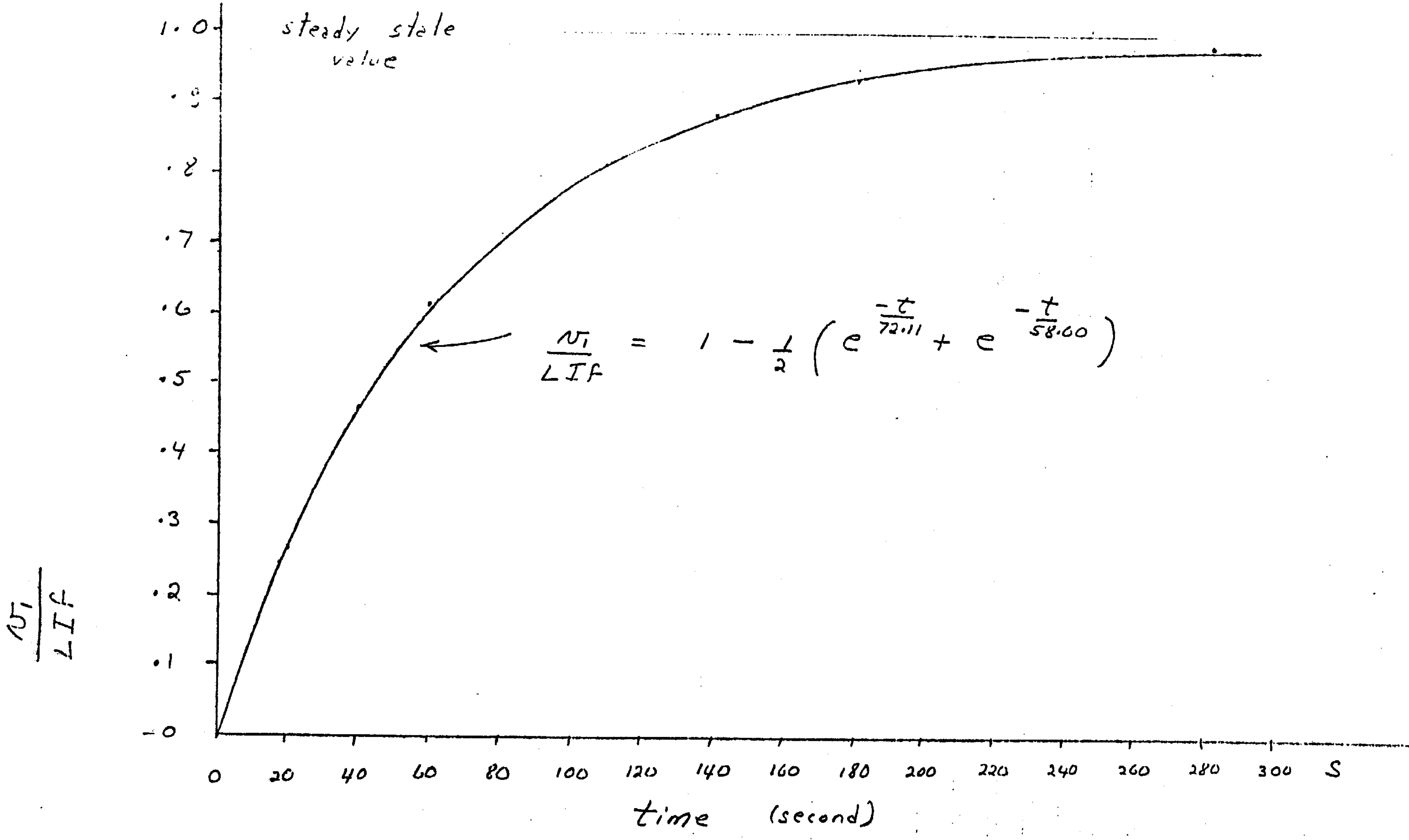
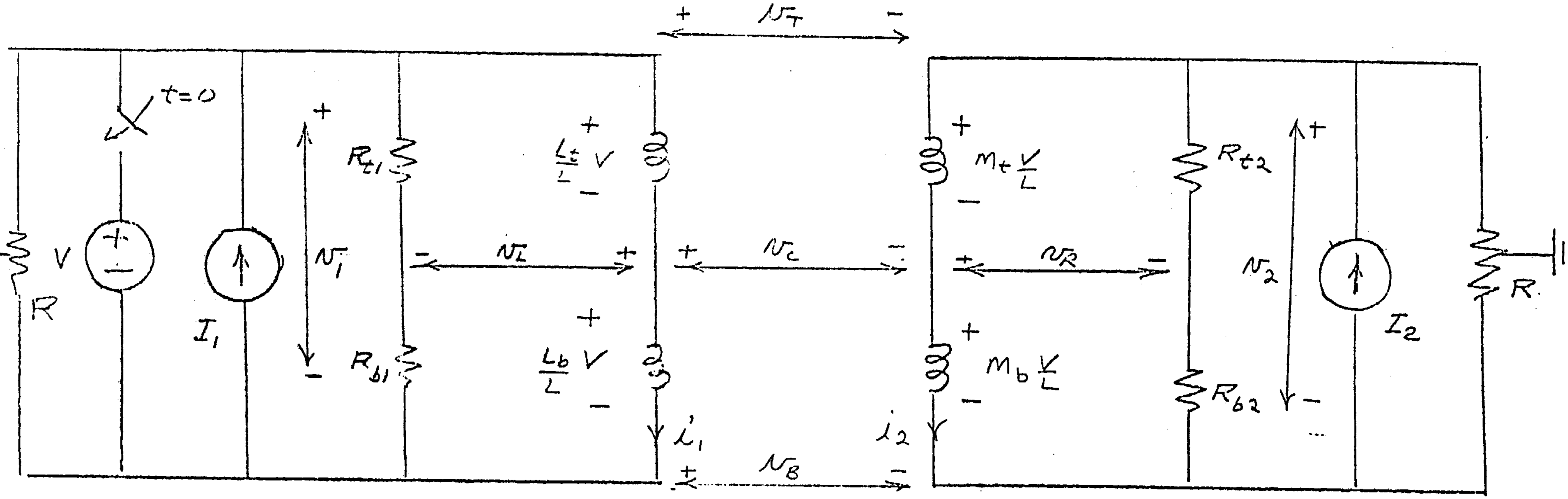


FIGURE 9

The transient voltage across the coil connected to a current source with a ramp change. The voltage approaches LIF as $t \to \infty$.



$I_1 = I$ $\qquad$ $i_1 = I + \frac{V}{L} t$ $\qquad$ $i_2 = I - \frac{KV}{R}$ $\qquad$ $I_2 = I$

$v_1 = V$ $\qquad$ $v_L = V\left(\frac{L_b}{L} - \frac{R_{b1}}{R_{B1}}\right)$ $\qquad$ $v_C - v_B = \frac{V}{L}(L_b - M_b)$

$v_2 = KV, \; K = \frac{M}{L}$ $\qquad$ $v_R = KV\left(\frac{M_b}{M} - \frac{R_{b2}}{R_{B2}}\right)$ $\qquad$ $v_C - v_T = \frac{V}{L}(M_t - L_t)$

FIGURE A1

The steady state voltages and currents resulting from the introduction of a voltage source across the left coil